Spherically Symmetric solutions in Multidimensional Gravity with the SU(2) Gauge Group as the Extra Dimensions

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The multidimensional gravity on the principal bundle with the SU(2) gauge group is considered. The numerical investigation of the spherically symmetric metrics with the center of symmetry is made. The solution of the gravitational equations depends on the boundary conditions of the "SU(2) gauge potential" (off-diagonal metric components) at the symmetry center and on the type of symmetry (symmetrical or antisymmetrical) of these potentials. In the chosen range of the boundary conditions it is shown that there are two types of solutions: wormhole-like and flux tube. The physical application of such kind of solutions as quantum handles in a spacetime foam is discussed.

I. INTRODUCTION

At the present time the most popular version of multidimensional (MD) gravity is a Kaluza-Klein gravity (for review see Ref. [1]) in which all space-like directions are equivalent, i.e. extra coordinates are the same as the space coordinates. Such approach is very natural but it has a great problem with the sharing[14] of extra dimensions (ED). In other words we should have some natural mechanism for the sharing the ED and space coordinates. It is well known that it is very difficult to realize such mechanism in the empty spacetime. It is necessary to introduce some external matter field for such sharing of ED. Of course such way kills the Einstein's idea that the matter can be effectively constructed from the pure geometry.

There is another possibility for the renewal of the above-mentioned Einstein's idea. We would like to return to the initial interpretation of Kaluza-Klein theory in which that all physical quantities should not depend on the ED. How it can be done? We offer to take the ED as a gauge group (for example, U(1), SU(2) or SU(3) and so on). The advantages of such an approach are obvious: (a) we will have the matter as the gauge fields; (b) the ED make up a symmetric space and therefore the physical fields will not depend on the extra coordinates. The first item (a) follows from the following theorem [2], [3]:

Let G be the group fibre of the principal bundle. Then there is a one-to-one correspondence between the G-invariant metrics G_{AB} (A, B are the multidimensional indices) on the total space $\mathcal{X}$ and the triples $(g_{\mu\nu}, A_\mu^a, \varphi)$.

$$ds^2 = G_{AB}dx^A dx^B = \varphi(x^\alpha) \sum_{a=5}^{\dim G} \left[\sigma^a + A_\mu^a(x^\alpha)dx^\mu \right]^2 + g_{\mu\nu}(x^\alpha)dx^\mu dx^\nu \tag{1}$$

here $g_{\mu\nu}$ is Einstein's pseudo - Riemannian metric on the base; A_μ^a is the gauge field of the group G (the off-diagonal components of the multidimensional metric); $dl^2 = \varphi(x^\mu)\sigma^a \sigma_a$ is the symmetric metric on the fibre.

This theorem tells us that the inclusion the off-diagonal components of the MD metric is equivalent to the inclusion gauge fields (U(1), SU(2) or SU(3)) and a scalar field $\varphi(x^\mu)$ which is connected with the linear size of the extra dimensions. These geometrical fields can act as the source of the exotic matter necessary for the formation of the

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wormhole's (WH) mouth. Such solutions were obtained in Ref's. [4] [5] [6] [7]. These solutions are spherically symmetric WH-like metrics[15] with finite longitudinal size. The throat of these WH-like solutions is located between two invariant surfaces on which $ds^2 = 0$. These results indicate that the exotic matter necessary for the formation of the WH mouth can appear in **vacuum multidimensional gravity** from the off-diagonal elements of the metric (the gauge fields) and from the metric on the fibre (the scalar field), rather than coming from some externally given exotic matter.

The second item (b) leads to the fact that all physical fields (scalar field φ, gauge fields A_μ^a and 4D metrical tensor $g_{\mu\nu}$) do not depend on the extra coordinates. Moreover the ED have an additional structure: every point is an element of a group (gauge group U(1), SU(2) or SU(3)), and **such structure cannot be destroyed by any perturbations.** This means that not any physical particle can penetrate to the ED. Otherwise it will destroy the symmetry of the ED and consequently an algebraic structure of the gauge group. In fact the gravity on the principal bundle give us the natural way for the compactification and sharing of the ED.

II. GRAVITY EQUATIONS FOR SU(2) GAUGE GROUP AS THE EXTRA DIMENSIONS

In this case the multidimensional (MD) spacetime is a total space of the principal bundle with SU(2) structural group. The fibre of this principal bundle (the ED) is SU(2) gauge group and the base is the 4D spacetime. The Langrangian is

$$L = \sqrt{-G}R \tag{2}$$

where G is the determinant and R is the Ricci scalar of the MD metric. According to above-mentioned theorem the MD metric on the total space is

$$ds^2 = \Sigma_{\bar{A}}\Sigma^{\bar{A}} \tag{3}$$

where

$$
\begin{aligned}
\Sigma^{\bar{A}} &= h_B^{\bar{A}} dx^B, & (4)\\
\Sigma^{\bar{a}} &= \varphi(x^\alpha)\left(\sigma^{\bar{a}} + A_\mu^{\bar{a}}(x^\alpha)dx^\mu\right), & (5)\\
\sigma^{\bar{a}} &= h_b^{\bar{a}} dx^b, & (6)\\
\Sigma^{\bar{\mu}} &= h_\nu^{\bar{\mu}}(x^\alpha)dx^\nu & (7)
\end{aligned}
$$

here x^B are the coordinates on the total space; $B = 0,1,2,3,5,6,7$ is the MD index; $\dim SU(2) = 3$; x^a is the coordinates on the group $SU(2)$ ($a = 5,6,7$); $x^\mu = 0,1,2,3$ are the coordinates on the base of the bundle; $\alpha, \mu, \nu = 0,1,2,3$; $\bar{A} = (\bar{\mu}, \bar{a})$ is the sieben-bein index; $h_B^{\bar{A}}$ is the N-bein; $\sigma^{\bar{a}}$ are the 1-forms on the group $SU(2)$ satisfying $d\sigma^{\bar{a}} = \epsilon_{\bar{b}\bar{c}}^{\bar{a}} \sigma^{\bar{b}} \sigma^{\bar{c}}$; $(\epsilon_{\bar{b}\bar{c}}^{\bar{a}})$ are the structural constants for the gauge group $SU(2)$; the signature of the MD metric is $\eta_{\bar{A}\bar{B}} = (+,-,-,-,-,-,-)$. We must note that the functions $\varphi, A_\mu^{\bar{a}}, h_\nu^{\bar{\mu}}$ can depend only on the x^μ points on the base as the fibres of our bundle are homogeneous spaces. Varying with respect to our physical degrees of freedom [7] leads to the following equations system

$$
\begin{aligned}
R_\mu^{\bar{A}} &= 0, & (8)\\
R_{\bar{a}}^{\bar{a}} = R_{\bar{5}}^{\bar{5}} + R_{\bar{6}}^{\bar{6}} + R_{\bar{7}}^{\bar{7}} &= 0. & (9)
\end{aligned}
$$

According to above-mentioned theorem the following dimensional reduction of the Ricci scalar $R(E)$ on the total space of the principal bundle ([3])

$$
\int d^4x d^d y \sqrt{|\det G_{AB}|}R(E) =
$$
$$
V_G \int d^4x \sqrt{|g|}\varphi^{d/2}\left[R(M) + R(G) - \frac{1}{4}\varphi F_{\mu\nu}^a F_a^{\mu\nu} + \frac{1}{4}d(d-1)\partial_\mu\varphi\partial^\mu\varphi\right] \tag{10}
$$

here $R(M), R(G)$ are the Ricci scalars of the base and structural group of the principal bundle respectively; V_G is the volume of the group $SU(2)$. The independent degrees of freedom in the gravity on the principal bundle are: scalar field $\varphi(x^\alpha)$, gauge potential $A_\mu^a(x^\alpha)$ and 4D metric tensor $g_{\mu\nu}(x^\alpha)$. The most important difference of this theory from the modern variants of the Kaluza - Klein gravity is that here we should vary on the φ but not with the every component of the metric on the ED (G_{55}, G_{66} and so on). This leads to essential decreasing of

the number of gravitational equations and to appearing of WH-like vacuum solutions which are necessary for the Einstein/Wheeler idea "mass without mass" and "charge without charge". For the ordinary 7D Kaluza-Klein theory with the Einstein equations $R_{AB} = \kappa T_{AB}$ we have $N = \frac{7 \cdot (7+1)}{2} = 28$ equations. In our case the number of Eq's(8), (9) are $N = (number\ of\ 4D\ Einstein\ eq's) + (number\ of\ Yang-Mills\ eq's) + (1\ eq.\ for\ scalar\ field) = 10+12+1 = 23$. Such essential simplification of the theory structure leads to the fact that in our case vacuum solutions (wormhole-like) appear which cannot exist in the ordinary Kaluza-Klein gravity (where we must destroy the primary Einstein idea about a geometrization of physics and insert a multidimensional matter).

Another remarkable peculiarity of such kind of gravitational theories is that the linear sizes of the ED can essentially differ from the Planck scale. It follows from the above-mentioned fact that not any physical test particle (and consequently not any physical body) can penetrate into the ED as far as it would destroy the algebraic (consequently symmetric) structure of the ED. Therefore the essential property of the gravity on the principal bundle is that *the linear size of the ED can be distinguished from the Planck scale.*

And finally in these gravitational theory does not appear the compactification problem because the ED are the compact gauge group: U(1), SU(2), SU(3) or another compact Lie group. This means that in our approach the compactification problem is not dynamical one and the compactness of the ED is inserted initially in such kind of the MD theory.

III.   METRIC ANSATZ AND INITIAL EQUATIONS

We will search a solution for the following 7D metric (here we follow to Ref. [7])

$$ds^2 = \frac{\Sigma^2(r)}{u^3(r)} dt^2 - dr^2 - a(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right) - R_0^2 u(r)\left(\sigma^a + A_\mu^a dx^\mu\right)^2 \tag{11}$$

here r_0 is some constant, σ^a ($a = 5, 6, 7$) are the Maurer-Cartan form with relation $d\sigma^a = \epsilon_{bc}^a \sigma^b \sigma^c$

$$\sigma^1 = \frac{1}{2}(\sin\alpha d\beta - \sin\beta\cos\alpha d\gamma), \tag{12}$$

$$\sigma^2 = -\frac{1}{2}(\cos\alpha d\beta + \sin\beta\sin\alpha d\gamma), \tag{13}$$

$$\sigma^3 = \frac{1}{2}(d\alpha + \cos\beta d\gamma), \tag{14}$$

where $0 \leq \beta \leq \pi, 0 \leq \gamma \leq 2\pi, 0 \leq \alpha \leq 4\pi$ are the Euler angles on the fibre. We choose the potential A_μ^a in the ordinary monopole-like form

$$A_\theta^a = \frac{1}{2}(1 - f(r))\{\sin\phi; -\cos\phi; 0\}, \tag{15}$$

$$A_\phi^a = \frac{1}{2}(1 - f(r))\sin\theta\{\cos\phi\cos\theta; \sin\phi\cos\theta; -\sin\theta\}, \tag{16}$$

$$A_t^a = v(r)\{\sin\theta\cos\phi; \sin\theta\sin\phi; \cos\theta\}, \tag{17}$$

Let us introduce the color electric E_i^a and magnetic H_i^a fields

$$E_i^a = F_{ti}^a, \tag{18}$$

$$H_i^a = \sqrt{\gamma}\epsilon_{ijk}\sqrt{g_{tt}}F^{ajk} \tag{19}$$

here the field strength components are defined via $F_{\mu\nu}^a = A_{\nu,\mu}^a - A_{\mu,\nu}^a + \epsilon_{bc}^a A_\mu^b A_\nu^c$, γ is the determinant of the 3D space matrix, $(i, j = 1, 2, 3)$ are the space index. In our case we have

$$E_r \propto v', \qquad E_{\theta,\phi} \propto vf, \tag{20}$$

$$H_r \propto \frac{\Sigma}{u^{3/2}}\frac{1 - f^2}{a}, \qquad H_{\theta,\phi} \propto f' \tag{21}$$

The substitution to the 7D gravitational equations (8) (9) leads to the following system of equations

$$\frac{\Sigma''}{\Sigma} + \frac{a'\Sigma'}{a\Sigma} - \frac{4}{R_0^2 u} - \frac{R_0^2 u}{4a} f'^2 - \frac{R_0^2 u}{8a^2} \left(f^2 - 1\right)^2 = 0,$$ (22)

$$24\frac{\Sigma' u'}{\Sigma u} - 24\frac{u'^2}{u^2} + 16\frac{a'\Sigma'}{a\Sigma} + 4\frac{a'^2}{2a^2} - \frac{16}{a} +$$
$$4\frac{R_0^2 u^4}{\Sigma^2} v'^2 - 2\frac{R_0^2 u}{a} f'^2 - 8\frac{R_0^2 u^4}{a\Sigma^2} f^2 v^2 + \frac{R_0^2 u}{a^2} \left(f^2 - 1\right)^2 - \frac{48}{uR_0^2} = 0,$$ (23)

$$\frac{a''}{a} + \frac{a'\Sigma'}{a\Sigma} - \frac{2}{a} + \frac{R_0^2 u}{4a} f'^2 - \frac{R_0^2 u^4}{a\Sigma^2} f^2 v^2 + \frac{R_0^2 u}{4a^2} \left(f^2 - 1\right)^2 = 0,$$ (24)

$$\frac{u''}{u} + \frac{u'\Sigma'}{u\Sigma} - \frac{u'^2}{u^2} + \frac{u'a'}{ua} - \frac{4}{R_0^2 u} +$$
$$\frac{R_0^2 u^4}{3\Sigma^2} v'^2 - \frac{R_0^2 u}{6a} f'^2 + \frac{2R_0^2 u^4}{3a\Sigma^2} f^2 v^2 - \frac{R_0^2 u}{12a^2} \left(f^2 - 1\right)^2 = 0,$$ (25)

$$v'' + v' \left(-\frac{\Sigma'}{\Sigma} + 4\frac{u'}{u} + \frac{a'}{a}\right) - \frac{2}{a} v f^2 = 0,$$ (26)

$$f'' + f' \left(\frac{\Sigma'}{\Sigma} + 4\frac{u'}{u}\right) + 4\frac{u^3}{\Sigma^2} f v^2 - \frac{f}{a} \left(f^2 - 1\right) = 0.$$ (27)

The equations (26) and (27) correspond to the ordinary "Yang-Mills" equations after the dimensional reduction.

In the consequence of the WH symmetry the functions $a(r), \Sigma(r)$ and $u(r)$ are symmetric and the functions $f(r)$ and $v(r)$ can be either symmetric or antisymmetric. Consequently we would like to consider the following different cases

1. the function $f(r)$ is symmetric and the function $v(r)$ is antisymmetric;

2. the function $f(r)$ is antisymmetric and the function $v(r)$ is symmetric;

3. both functions $f(r)$ and $v(r)$ is antisymmetric;

4. both functions $f(r)$ and $v(r)$ is symmetric.

The first case was considered in the Ref. [7]. The result is that there are three type of solutions : wormhole-like, infinite and finite flux tubes solutions.

IV.   BOUNDARY CONDITIONS FOR THE 4D METRIC

Now we will write the initial conditions for the 4D metric functions. At the center of symmetry of the WH ($r = 0$) we can expand functions $a(r), \Sigma(r)$ and $u(r)$ by this manner

$$a(x) = 1 + \frac{a_2}{2} x^2 + \cdots,$$ (28)

$$\Sigma(x) = \Sigma_0 + \frac{\Sigma_2}{2} x^2 + \cdots,$$ (29)

$$u(x) = u_0 + \frac{u_2}{2} x^2 + \cdots,$$ (30)

here we introduce the dimensionless coordinate $x = r/\sqrt{a_0}$ ($a_0 = a(0)$) and redefine $a(x)/a_0 \to a(x)$, $\sqrt{a_0} v(x) \to v(x)$, $R_0^2/a_0 \to R_0^2$. Then we can rescale time and the constant R_0 so that $\Sigma_0 = u_0 = 1$. Thus we have the following initial conditions for the functions $a(r), \Sigma(r)$ and $u(r)$ for the numerical calculations

$$a_0 = 1, \qquad u_0 = 1, \qquad \Sigma_0 = 1,$$ (31)
$$a_0' = 0, \qquad u_0' = 0, \qquad \Sigma_0' = 0.$$ (32)

V.   FUNCTION $f(r)$ IS ANTISYMMETRIC AND FUNCTION $v(r)$ IS SYMMETRIC

In this case we have the following expansion of functions $f(x)$ and $v(x)$ at the origin

$$v(x) = v_0 + \frac{v_2}{2}x^2 + \cdots, \tag{33}$$

$$f(x) = f_1 x + \frac{f_3}{6}x^3 + \cdots, \tag{34}$$

this equation is written in dimensionless variables. The constrained equation (23) for the initial data give us

$$R_0^2 = 4\frac{2 + \sqrt{7 - 6f_1^2}}{1 - 2f_1^2} \tag{35}$$

As $R_0^2 > 0$ we have the following constraint for f_1 : $|f_1| < 1/\sqrt{2}$. The numerical calculations are presented on the Fig's (2), (3), (4), (5), (6).

In this case the numerical calculations show us that the $a(x)$ function is monotonically decreasing one and consequently its behaviour is defined with the value of $a_0''(0)$. Eq. (24) give us the following expression

$$\frac{a_0''}{a_0} = 2 - \left(1 + f_1^2\right)\frac{2 + \sqrt{7 - 6f_1^2}}{1 - 2f_1^2} \tag{36}$$

On the Fig.(1) this curve is shown. As a result we see that $a_0'' < 0$ for all f_1 and consequently it confirms our numerical investigation that $a(x)$ is monotonically decreasing function.

VI.   BOTH FUNCTIONS $f(r)$ AND $v(r)$ ARE ANTISYMMETRIC

In this case we have the following expansion of functions $f(x)$ and $v(x)$ at the origin

$$v(x) = v_1 x + \frac{v_3}{6}x^3 + \cdots, \tag{37}$$

$$f(x) = f_1 x + \frac{f_3}{6}x^3 + \cdots, \tag{38}$$

this equation is written in dimensionless variables. The constrained equation (23) for the initial data give us

$$r_0^2 = 4\frac{2 + \sqrt{7 + 12v_1^2 - 6f_1^2}}{1 + 4v_1^2 - 2f_1^2}. \tag{39}$$

The positivity condition of R_0^2 give us

$$2f_1^2 - 4v_1^2 < 1 \tag{40}$$

In the chosen range of the parameters (f_1, v_1) the numerical calculations lead to the fact that on the (f_1, v_1) plane there are two regions with the different solutions type. On Fig.(7) these regions with the different type of solutions are shown.

The numerical calculations are presented on the Fig's (8), (9), (10), (11), (12). We see that there is two type of solutions. The first type with decreasing $u(x)$ we can name as a WH-like solutions. For this type of solutions we have the following condition $a_0'' > 0$. With an accuracy of the numerical calculations we can say that there is a point x_1 for which $u(\pm x_1) = 0$. Probably in these points $ds^2 = 0$ that is similar to the 5D case investigated in Ref.[6]. We intend to investigate in more details the behavior of the metric close to such points with the help of approximate analytical methods in the future. The second type satisfies the condition $a_0'' < 0$ and with an accuracy of the numerical calculations there is a point x_1 for which $a(\pm x_1) = 0$. With great probability we have singularities in these two points, *i.e.* such solutions are like to flux tube of color "electric" and "magnetic" fields between two singularities. It is interesting to note that this type of solutions is very similar to the confinement mechanism in QCD where two quarks are located at the ends of a flux tube with color electric and magnetic fields running quark and antiquark.

VII. BOTH FUNCTIONS $f(r)$ AND $v(r)$ ARE SYMMETRIC

In this case we have the following expansion of functions $f(x)$ and $v(x)$ at the origin

$$v(x) = v_0 + \frac{v_2}{2} x^2 + \cdots, \tag{41}$$

$$f(x) = f_0 + \frac{f_2}{2} x^2 + \cdots, \tag{42}$$

this equation is written in dimensionless variables. The constrained equation (23) for the initial data give us

$$R_0^2 = 4 \frac{2 + \sqrt{4 + 3\left[\left(f_0^2 - 1\right)^2 - 8f_0^2 v_0^2\right]}}{\left(f_0^2 - 1\right)^2 - 8f_0^2 v_0^2} \tag{43}$$

The positivity condition of R_0^2 gives us

$$\left(f_0^2 - 1\right)^2 > 8 f_0^2 v_0^2 \tag{44}$$

In this case the numerical calculations show us that the $a(x)$ function is monotonically decreasing one and consequently its behavior is defined with the value of $a_0''(0)$. Eq's (23) (24) give us the following expression

$$a_0'' = -\frac{r_0^2}{8} \left(f_0^2 - 1\right)^2 - \frac{6}{r_0^2} < 0. \tag{45}$$

As a result we see that $a_0'' < 0$ for all f_0, v_0 and consequently it confirms our numerical investigation that $a(x)$ is monotonically decreasing function. The qualitative behavior of all functions $a(x), \Sigma(x), u(x), f(x)$ and $v(x)$ are similar with the second case (where $f(x)$ is antisymmetric and $v(x)$ is symmetric one).

VIII. DISCUSSION

On the basis of Ref.[6] and the investigations presented here we can say that the spherically symmetric solutions of the gravity on the principal bundle with SU(2) gauge group as the structural group (extra dimensions) and which are symmetric regarding to some hypersurface $r = const$ (in our case $r = 0$) are presented by three types :

1. Wormhole - like solutions. These solutions are characterized that $a(0) = minimum$ and there are two hypersurfaces by $r = \pm r_0$ where $ds^2 = 0$.

2. Infinite flux tube. This solution was presented in Ref.[8]. It has $a(r) = const$ ($r \in (-\infty, +\infty)$) and color electric and magnetic fields are parallel.

3. Finite flux tubes. These solutions have two singularities at $r = \pm r_0$ and function $a(r)$ has a maximum at $r = 0$.

The difference between these possibilities are determinated by the symmetry of $f(r)$ and $v(r)$ functions and their boundary conditions $f(0), f'(0), v(0)$ and $v'(0)$. It means that the type of solutions is completely defined by color electric and magnetic fields at the symmetry center of the metric.

Now we would like to compare our designations for the solutions (wormhole and flux tube) with other similar definitions.

A. Wormholes

According to Ref.[9] the wormhole is time independent (in the simplest case), nonrotating, and spherically symmetric bridge between two universes. The 4D spacetime metric can be put into the form

$$ds^2 = \frac{dt^2}{\Delta(r)} - dr^2 - a(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{46}$$

that can be compared with 4D part of Eq.(11). The WH should have the following properties :

- The coordinate $r \in (-\infty, +\infty)$.

- The event horizon is absent.

- There is a throat where $a(0) = minimum$.

- There are two asymptotically flat regions at $r = \pm\infty$.

Comparing these definitions with our calculations we can see why we called our first type of solutions as *wormhole-like* solutions :

- The coordinate $r \in (-r_0, +r_0)$. It is possible that similar to the 5D wormhole-like solutions we can continue these solutions up to $r \in (-\infty, +\infty)$. In this case the metric signature will be changed : from $(+, -, -, -, -)$ by $r \in (-r_0, +r_0)$ to $(-, -, -, -, +)$ by $r \in (-\infty, -r_0)$ and $r \in (+r_0, +\infty)$. Such situation for 5D case was discussed in Ref.[10]. Following to Bronnikov [11] we can name two hypersurfaces $r = \pm r_0$ where $ds^2 = 0$ and such exchange of the metric signature takes place as $T-$horizons.

- There are two $T-$horizons.

- There is a minimum of $a(r)$.

- The existence of asymptotically flat regions is not yet unknown but the comparison with the 5D case give us a hope that such regions exist.

Thus the distinction and similarities between these standard and our metrics is evident. The most important peculiarity of presented here wormhole-like solutions is that they are vacuum solutions without any matter. The color electric and magnetic SU(2) gauge fields are the off-diagonal components of 7D metric.

B. Euclidean wormholes

Euclidean wormholes have the topology $R \times S^{n-1}$, where R is an imaginary time and S^{n-1} is $(n-1)$ dimensional sphere. They are commonly thought of as "instantons" in the gravitational field. The distinction with our case is evident.

IX. FLUX TUBES

The notion of flux tube has arisen in the quantum chromodynamics where this one is a hypothesized tube filled with gauge field and stretched between quark-antiquark. Such field configuration arises in the consequence of a specific non-linear potential in the SU(3) Yang-Mills theory.

In the gravity can be some types of spacetime with a finite flux of electric/magnetic fields.

- Melvin type of solutions (see, for example, Melvin Universe [12]). The $t = const$ spacelike section has the cylindrical symmetry and filled with cylindrically symmetric magnetic field which has a finite flux of this field.

- The modern multidimensional Kaluza-Klein theories have solutions which can contain branes with some generalization of electric field and finite flux of this fileds, *i.e.* flux branes.

- Levi-Civita-Betotti-Robinson solutions [13]. These solutions can be multidimensional [8]. 4D part of these metrics has the following topologies : $R \times I \times S^2 = (time) \times (radial\ coordinate) \times (sphere)$. The range of the radial coordinate can be finite $I = [-r_0, +r_0]$ and infinite $I = R = (-\infty, +\infty)$. Roughly speaking these solutions have a finite cross section (S^2). They are filled with electric/magnetic field with finite flux of these fields. We have shown that such solutions exist in our case and the difference between wormhole-like and flux tube solutions is defined by the relation between electric and magnetic fields at the center of symmetry of this metric.

X. CONCLUSIONS

In this paper we have shown (with an accuracy of the numerical calculations) that the spherically symmetric metric of the MD gravity on the principal bundle with SU(2) gauge group as the ED are either WH-like or flux tube metrics. The numerical calculations indicate that the type of solutions is determined by a relation between the color "electric" $(E_{r,\theta,\varphi})$, "magnetic" $(H_{r,\theta,\varphi})$ fields, the scalar curvatures of the ED (R_{ED}) and the sphere S^2 (R_{S^2}) at $r = 0$. Let us write the constraint equation (23)

$$\frac{u_0^3}{\Sigma_0^2}v_0'^2 + \frac{1}{4a_0^2}\left(f_0^2 - 1\right)^2 - \frac{1}{2a_0}f_0'^2 - 2\frac{u_0^3}{a_0\Sigma_0^2}f_0^2v_0^2 - \frac{2}{r_0^2u_0}\left(\frac{2}{a_0} + \frac{6}{u_0r_0^2}\right) =$$
$$\left(E_r^2\right)_0 + \frac{1}{4}\left(H_r^2\right)_0 - 2\left(E_\theta^2\right)_0 - \frac{1}{2}\left(H_\theta^2\right)_0 - \frac{2}{r_0u_0}\left(R_{S^2} + R_{ED}\right) = 0. \tag{47}$$

Immediately we see that the necessary condition for the existence of the WH-like solutions is that it must be a flux of color "electric" $\Phi_E = 4\pi a_0 E_r^2$ and "magnetic" fields $\Phi_H = 4\pi a_0 H_r^2$ through the WH mouth. Only in this case Eg. (23) is fulfilled. The numerical calculations confirm such conclusion: only for the first and the third cases with $E_r \propto f_0' = f_1 \neq 0$ we will have the WH-like solutions.

The WH-like solutions can have very interesting physical consequences. Let us imagine that the universe is separated into the regions with dynamical and nondynamical metric on the ED [7]. It means that in the region with nondynamical $G_{\bar{a}\bar{a}}$ $(\bar{a} = 5, 6, 7)$ metric components we have 4D gravity interacting with the SU(2) gauge field and we have not an equation for the scalar field φ $(\varphi(x^\mu) = const)$ (this is the Kaluza - Klein gravity in its initial interpretation). In the region with dynamical $G_{\bar{a}\bar{a}}$ we have the full MD gravity. Such situation can be realized on the level of the spacetime foam, $i.e.$ the quantum handles of spacetime foam are the regions with the dynamical $G_{\bar{a}\bar{a}}$ and the region between these quantum handles is the region with nondynamical $G_{\bar{a}\bar{a}}$. In this situation each single quantum handle (wormhole) is the above-mentioned WH-like solution attached to an exterior spacetime on the surfaces where $u(\pm r_1) = 0$ is fulfilled.
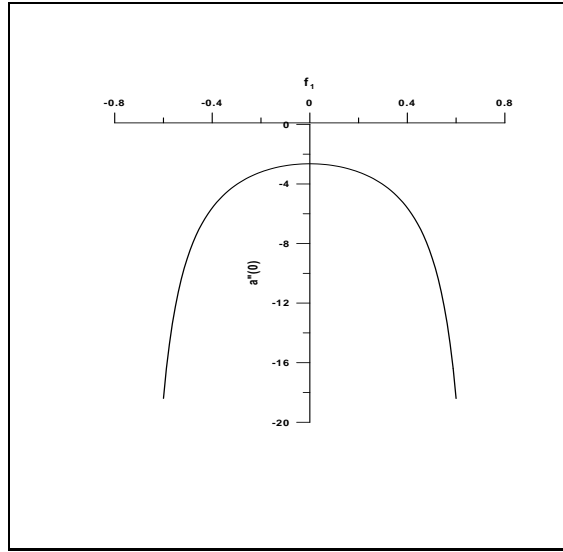
FIG. 1: The case 2 : the function $f(r)$ is antisymmetric and $v(r)$ is symmetric. a_0'' as a function on f_1.
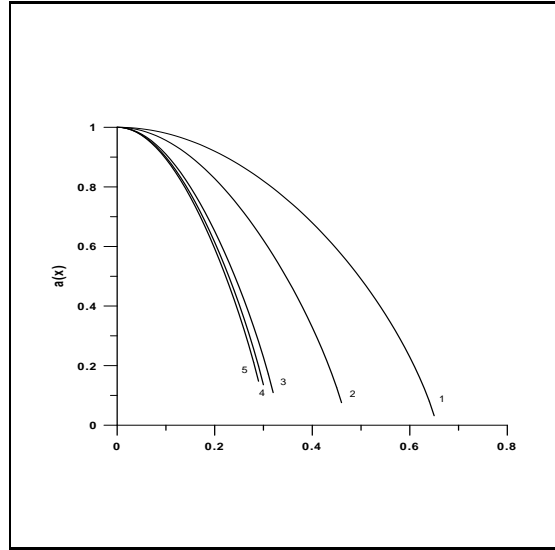

FIG. 2: The case 2 : the function $a(x)$. $v_0 = 0.2$; $f_1 = 0.3,\ 0.5,\ 0.6,\ 0.61,\ 0.615$

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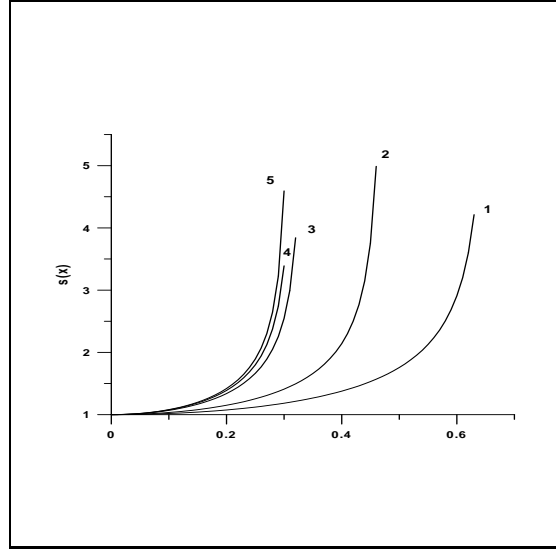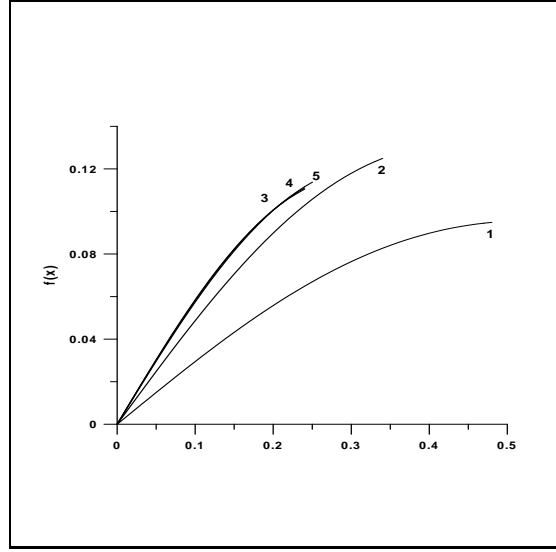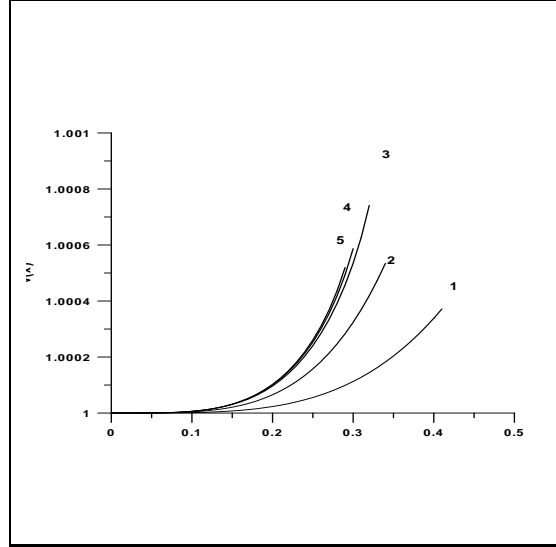
FIG. 3: The case 2 : the function $s(x)$. $v_0 = 0.2$; $f_1 = 0.3,\ 0.5,\ 0.6,\ 0.61,\ 0.615$
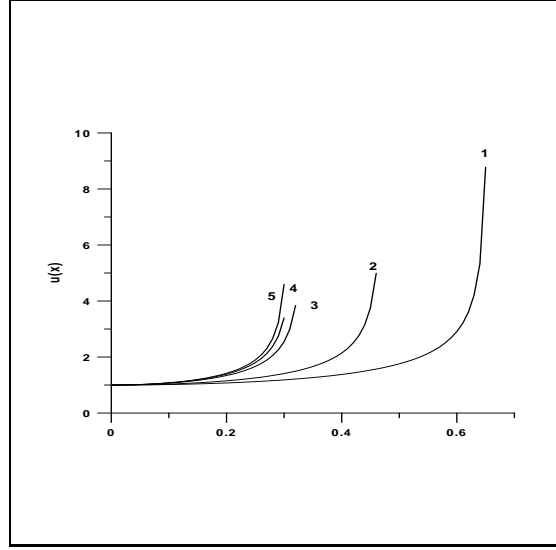
FIG. 4: The case 2 : the function $u(x)$. $v_0 = 0.2$; $f_1 = 0.3,\ 0.5,\ 0.6,\ 0.61,\ 0.615$

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[14] This means: a mechanism should exist in Nature which does some dimensions observed for us and remaining extra dimensions unobserved at least for the modern experimental physics

[15] The difference between ordinary wormhole and wormhole-like metrics will be discussed in section VIII

FIG. 5: The case 2 : the function $f(x)$. $v_0 = 0.2$; $f_1 = 0.3$, 0.5, 0.6, 0.61, 0.615

FIG. 6: The case 2 : the function $v(x)$. $v_0 = 0.2$; $f_1 = 0.3$, 0.5, 0.6, 0.61, 0.615
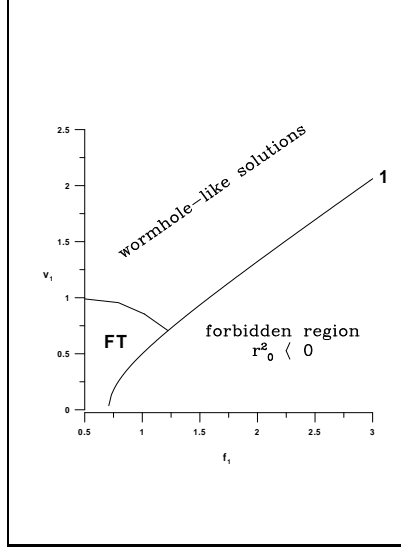
FIG. 7: The case 3 : both functions $f(r)$ and $v(r)$ are antisymmetric. The **FT** region is the region with the flux tube solutions. $r_0^2 < 0$ for the region under curve 1
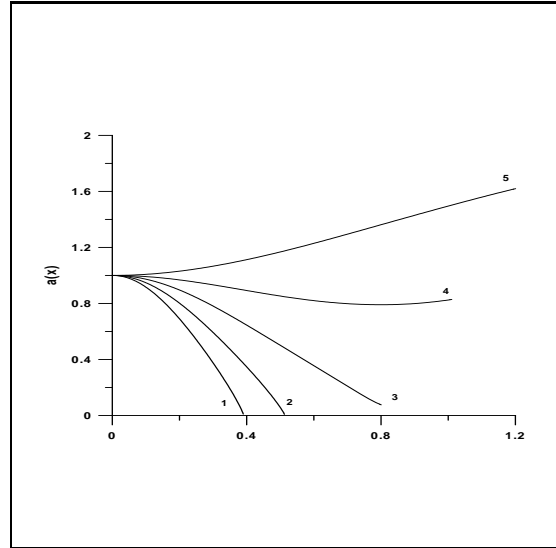

FIG. 8: The case 3 : the function $a(x)$. $f_1 = 1.0$; $v_1 = 0.6$, 0.65, 0.75, 1.0, 5.0 according to the curves 1, 2, 3, 4, 5.
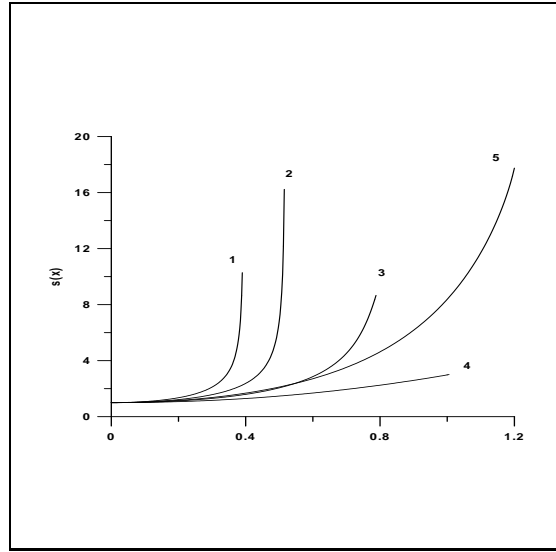
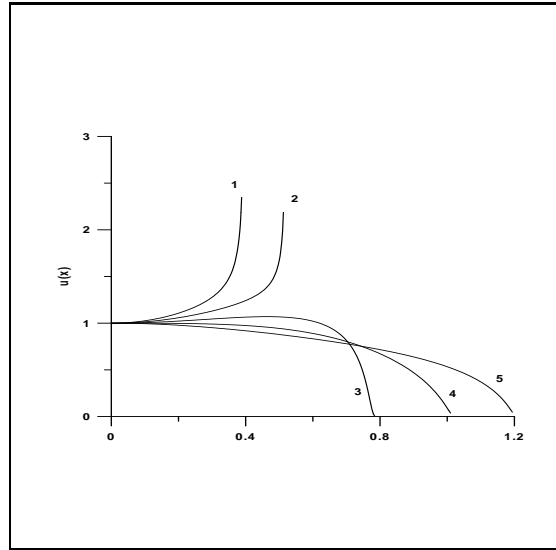FIG. 9: The case 3 : the function $s(x)$. $f_1 = 1.0$; $v_1 = 0.6,\ 0.65,\ 0.75,\ 1.0,\ 5.0$

FIG. 10: The case 3 : the function $u(x)$. $f_1 = 1.0$; $v_1 = 0.6,\ 0.65,\ 0.75,\ 1.0,\ 5.0$
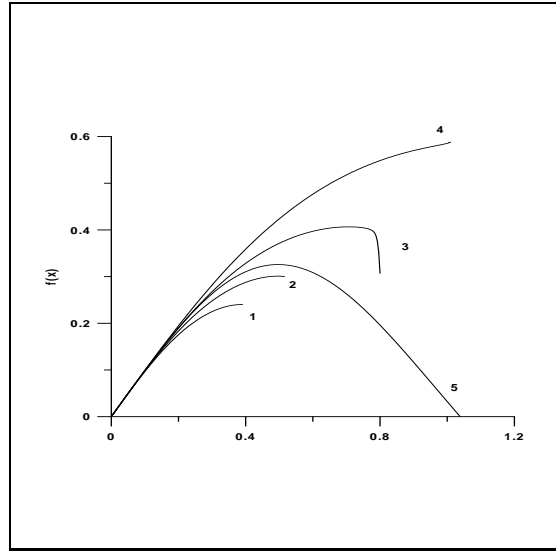
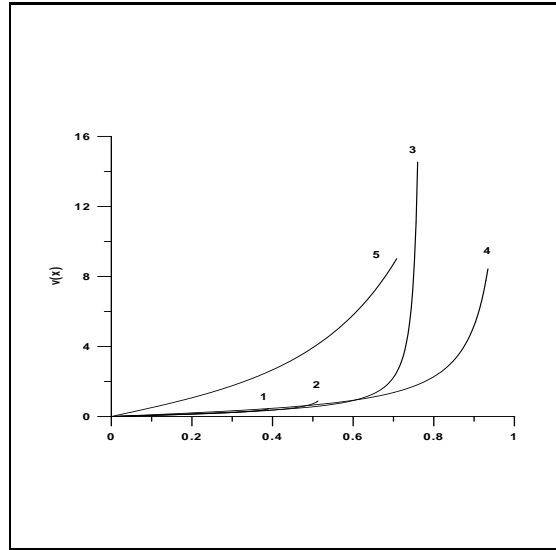FIG. 11: The case 3 : the function $f(x)$. $f_1 = 1.0$; $v_1 = 0.6$, 0.65, 0.75, 1.0, 5.0

FIG. 12: The case 3 : the function $v(x)$. $f_1 = 1.0$; $v_1 = 0.6$, 0.65, 0.75, 1.0, 5.0